ON GEOMETRICAL DEMONSTRATION

(*On the Geometrical Mind*)

1. We may have three main objects in the study of truth: first, to find it when we are seeking it; second, to demonstrate it after we have found it; third, to distinguish it from error by examining it.

I shall not speak of the first; I shall deal especially with the second, which includes the third. For if we know how to prove the truth, we shall know at the same time how to distinguish it from error, since by examining whether the proof we give conforms to the rules we know we shall find out whether the truth has been exactly demonstrated.

Geometry, which excels in all three of these ways, has expounded the art of finding unknown truths. This art, which is called analysis, would be useless to discuss after all the excellent books that have been written about it.

The art of demonstrating truths already found and of making them so clear that their proofs cannot be refuted is the only art I wish to give. To that end I have only to expound the method used by geometry, which geometry itself teaches perfectly by example without ever putting it into words. And because this art consists of two main parts, the proving of each proposition individually and the arranging of all the propositions in the best order, my treatise will have two sections, of which one will contain the rules of geometrical demonstrations, that is, scientific and perfect demonstrations, and the other will contain the rules of geometrical order, that is, scientific and complete order; so that the two sections taken together will include everything necessary for guiding the reason in proving truths and in distinguishing them from errors. My intention is to give these rules in their entirety.

SECTION 1. *Concerning the method of geometrical demonstrations, that is, scientific and perfect demonstrations*

I can give no clearer idea of the procedure we should follow to make our demonstrations convincing than by expounding the method observed in geometry.

But first I must give the idea of a method still more eminent and complete, but a method to which man could never attain. For what goes beyond geometry goes beyond man. Nevertheless I must say something about it although it is impossible to put it into practice.

This true method, which would produce demonstrations of supreme excellence if it were possible to attain to it, would consist of two main rules: to use no term of which we had not already clearly explained the meaning, and never to put forward any proposition unless demonstrated by truths already known; that is, in a word, to define all the terms and to prove all the propositions. But to follow the very order I am expounding I must state what I understand by definition.

Geometry recognizes only those definitions which logicians call nominal, impositions of a name, that is, on things clearly designated by terms perfectly well known. I speak of such definitions only.

Their usefulness and function is to clarify discourse and to make it more concise by expressing by the name we impose what would otherwise require several terms, in such a way however that the name imposed remains deprived of every other meaning, if it has any, keeping only that one to which it has been uniquely assigned. For example, if we need to distinguish among the numbers those which can be divided into two equal parts from those which cannot, in order to avoid the frequent repetition of this condition we give it a name as follows: I call every number that can be divided into two equal parts an even number.

This is a geometrical definition, because after a thing, in this case every number that can be divided into two equal parts, has been clearly pointed out, it is given a name emptied of all other meaning, if it has any, that it may take the meaning of the thing pointed out.

Whence it is evident that there is great freedom of definition and that definitions are never subject to contradiction, for nothing is more permissible than to give whatever name we please to a thing we have clearly pointed out. Only we must be careful not to take advantage of our freedom to impose names by giving the same name to two different things.

Not that it is inadmissible to do so if we avoid confusion by not extending the consequences of one to the other.

But if we fall into this vice, we can apply to it a most sure and infallible remedy, which is to substitute in the mind the definition in place of the thing defined and always to have such an awareness of the definition that every time we speak, for example, of an even number, we mean precisely a number that can be divided into two equal parts, and that we understand these two things as being so inseparably joined in thought that as soon as one of them occurs in discourse the mind at once attaches the other to it. For geometers and all those who proceed scientifically impose names on things only for concision of discourse and not to impoverish or alter the idea of the subjects of discourse. And they expect the mind always to supplement with the whole definition the short terms, which they use only to avoid the confusion caused by a multitude of words.

Nothing acts more quickly and more effectively against the surprise attacks of captious sophists than this method, which we must always have ready for use, and which alone suffices to banish every kind of difficulty and equivocation.

With these things well understood, I return to my account of the true order, which consists, as I was saying, in defining everything and in proving everything.

Certainly this method would be beautiful, but it is absolutely impossible. For it is evident that the first terms we wished to define would presuppose others for their explication, and that similarly the first propositions we wished to prove would suppose others that preceded them. And thus it is clear we should never arrive at the first propositions.

Accordingly, as we proceed ever further with our investigations, we come of necessity to primitive words which can no longer be defined and to principles so clear that it is no longer possible to find others more clear for their demonstration. Whence it is apparent that men are naturally and inevitably powerless to deal with any science whatsoever in an absolutely perfect order.

But it does not follow that we should abandon every kind of order. For there is one, the order of geometry, which is indeed inferior in that it is less convincing but not in that it is less certain. It does not define everything and does not prove everything, and in this it yields to the perfect order. But it assumes only things clear and invariant by the natural light, and that is why it is perfectly true, nature supporting it in default of discourse. This order, the most perfect possessed by man, does not consist either in defining everything and proving everything or in defining nothing and proving nothing; but it consists in holding to the mean of not defining things clear and understood by all men and defining all the rest, of not proving all the things known by men and proving all the rest. They sin equally against this order who undertake to define everything and prove everything and who fail to define and prove those things which are not self-evident.

This is perfectly illustrated by geometry. It does not define space, time, motion, number, equality, or their like, which are very numerous, because these terms point out so naturally the things they signify to those who understand the language that whatever clarification we might want to give would contribute more obscurity than instruction. For nothing is more ineffectual than the discourse of those who try to define these primitive words. What need is there, for example, to explain what we mean by the word *man*? Are we not sufficiently aware of what the thing is we wish to designate by this term? And what help did Plato think he was giving us by saying that man was a two-legged animal without feathers? As if the idea of man I naturally have and cannot express were not clearer and more certain than that given me by his useless and even ridiculous explanation, since a man by losing his two legs does not lose his humanity and a capon by losing his feathers does not take on humanity.

There are even those who go to the absurdity of explaining a word by the word itself. I know of some who have defined light in this way: "Light is a luminary motion of luminous bodies," as if we could understand the words *luminary* and *luminous* without understanding the word *light* [*lumière*].

We cannot undertake to define being without falling into this absurdity, for we cannot define any word without beginning with these words *it is*, either explicitly or implicitly. Therefore to define being we would have to say *it is*, and thus use the word defined in the definition.

It is sufficiently clear from this that there are words incapable of definition. And if nature had not made up for this defect by giving a like idea to all men, all our expressions would be confused; whereas we make use of them with the same assurance and the same certainty we should have if they had been explained in a perfectly unambiguous way, because nature itself has given us, without words, a clearer understanding of them than we gain through art with all our explanations.

Not that all men have the same idea of the essence of those things I say it is impossible and useless to define.

For, to take an example, time is of this kind. Who can define it? And why try, since all men know what we mean when we speak of time, without further designation? There are nonetheless many different opinions as to the essence of time. Some say that it is the motion of a created thing; others, the measure of motion, etc. Therefore it is not the nature of these things that I say is known by all; it is simply the relation between the name and the thing, so that at the expression *time* everyone considers the same object (which is sufficient to make it unnecessary to define this term) although afterwards, looking into the nature of time, we may come to have different opinions when we have set ourselves to thinking about it. For definitions are made only to point out the things named and not to reveal their nature.

Not that we may not call by the name *time* the motion of a created thing, for, as I was just saying, in nothing is there more freedom than in definitions. But as a result of this definition there will be two things called by the name *time*: one is what everybody naturally understands by this word and what all who speak our language name with this term; the other will be the motion of a created thing, for this too will be called by that name in accordance with the new definition. We shall have, therefore, to avoid ambiguity and to keep the consequences separate. For it will not follow that what we naturally understand by the word *time* is in fact the motion of a created thing. We were free to give these two things the same name, but we shall not therefore be free to make them coincide in nature as well as in name.

Thus if the proposition is put forward: "Time is the motion of a created thing," we must ask what is meant by the word *time*, whether, that is, it keeps its usual and commonly accepted meaning or whether it has been emptied of that meaning to receive for this occasion, as its meaning, the motion of a created thing. If it has been deprived of all other meaning, the statement cannot be contradicted, and it will be an arbitrary definition with the result, as I have said, that there will be two things having the same name. But if it has been allowed to keep its usual meaning and nevertheless it is maintained that what this word means is the motion of a created thing, the statement can be contradicted. It is no longer an arbitrary definition; it is a proposition requiring proof unless it is evident of itself, in which case it will be a principle and an axiom but never a definition, because in this statement it is not understood that the word *time* signifies the same thing as the words, the *motion of a created thing*, but it is understood that what is meant by the term *time* is this supposed motion.

If I had not known how necessary it is that this should be perfectly understood and how, in familiar speech as in that of science, occasions like the one I have given as an example constantly arise, I should not have lingered over it. But it seems to me, from my experience of the confusion of disputes, that it is impossible to go too thoroughly into that exactness of mind for the sake of which I am writing this whole treatise more than for the subject I deal with in it.

For how many people are there who think they have defined time when they have said that it is the measure of motion, leaving it meanwhile its usual meaning!

And yet they have made a proposition and not a definition. How many are there likewise who think they have defined motion when they have said: *Motus nec simpliciter actus nec mera potentia est, sed actus entis in potentia!* And nevertheless if they let the word *motion* keep its usual meaning, as they do, it is not a definition but a proposition; and in this way confounding the definitions they call nominal, which are the true definitions, arbitrary, permissible, and geometrical, with those they call real, which are really propositions by no means arbitrary but subject to contradiction, they take the liberty of making the latter as well as the former; and each defining the same things in his own fashion by a freedom which is as forbidden in this kind of definition as it is permitted in the first, they mix up everything, and losing all order and all light, they lose themselves and wander in a maze of difficulties inexplicable.

We shall never get into such trouble if we follow the order of geometry. That wise science is very far from defining such primitive words as space, time, motion, equality, majority, decrease, all, and those others which the generality of men understand without explanation. But with the exception of these the remaining terms used by geometry are so clarified and defined that we have no need of a dictionary to understand any one of them, so that in a word all these terms are perfectly intelligible either by the natural light or by the definitions given.

This is the way geometry avoids all those vices which may be encountered in connection with the first point, which is to define only those things that need to be defined. It observes the same conduct with respect to the second point, which is to prove those propositions that are not evident. For when geometry has reached the first truths that can be known, it stops there and requires that they be granted since it has nothing clearer to prove them with; so that all the propositions of geometry are perfectly demonstrated either by the natural light or by proofs.

Whence it is that if this science does not define and demonstrate everything, it is only because it is impossible for us to do so. But since nature supplies everything not given by the science, the order of that science, though it does not give a superhuman perfection, has all the perfection men are capable of. It seemed to me fitting at the very beginning of the treatise to give this. . . .

Perhaps it will be thought strange that geometry cannot define any of the things that are its principal objects; for it can define neither motion nor numbers nor space, and yet these are the three things it particularly considers and in accordance with whose investigation it takes the three different names of mechanics, arithmetic, geometry, this last word belonging both to the genus and to the species.

But we shall feel no surprise if we observe that, this admirable science concerning itself only with the simplest things, the very quality which makes them worthy of being its objects makes them incapable of definition; so that the lack of definition is rather a perfection than a defect because it does not come from their obscurity but on the contrary from their extreme evidence, which is such that although it is not so convincing as a demonstration, it is fully as certain. Geometry supposes then that we know what thing is meant by the words: *motion*, *number*, *space*; and without stopping uselessly to define them it penetrates their nature and lays bare their marvelous properties.

These three things, which comprise the entire universe, in accordance with the words: *Deus fecit omnia in pondere, in numero et mensura*, are reciprocally and necessarily related. For we cannot imagine a motion without something which moves, and this thing being one, that unity is the origin of all number. Finally, since motion is impossible without space, we see that these three things are contained in the first. Even time is included there too, for motion and time are correlative (fast and slow, which differentiate motion, having a necessary reference to time).

Thus there are properties common to everything, the knowledge of which opens the mind to the greatest marvels of nature.

The most important is constituted by the two infinites which are found everywhere, the infinitely great and the infinitely small.

For however fast a motion may be, we can conceive a faster, and make that still faster, and thus forever to infinity without ever reaching a motion so fast that we can no longer add to it. And on the contrary, however slow a motion may be, we can make it slower, and that still slower, and so to infinity without ever reaching such a degree of slowness that we cannot still descend to an infinity of lower degrees without falling into rest.

Similarly, however great a number may be, we can conceive a greater, and again a greater than that, and so to infinity without ever reaching one which can no longer be increased. And on the contrary, however small a number may be, such as 1/100 or 1/10,000, we can still conceive a smaller, and keep on to infinity without reaching zero or nothing.

However great a space may be, we can conceive a greater, and again a still greater, and so to infinity without ever reaching one which can no longer be increased. And on the contrary, however small a space may be, we can still consider a smaller, and keep on to infinity without ever reaching an indivisible space which no longer has any extension.

It is the same with time. We can always conceive a greater without a last, and a lesser without reaching an instant, a pure nothing of duration.

Which is saying, in a word, that whatever motion, whatever number, whatever space, whatever time we take, there is always a greater and a lesser, so that they all keep between nothing and infinity, being always infinitely distant from these extremes.

None of these truths can be demonstrated, and yet they are the foundations and the principles of geometry. But since the cause that makes them incapable of demonstration is not their obscurity but on the contrary their extreme evidence, this lack of proof is not a defect but rather a perfection.

Whence we see that geometry can neither define its objects nor prove its principles, but for the sole and favorable reason that both have by nature an extreme clarity, which is more powerfully convincing to reason than any discourse.

For what is more evident than this truth, that any number whatsoever can be increased? Can it not be doubled? That the speed of a motion can be doubled, and that a space likewise can be doubled? And also who can doubt that any number whatsoever can be divided in half, and its half again divided in half? For could that half be nothing? And how could these two halves, which would be two zeros, make a number? In the same way cannot the speed of a motion, however slow, be cut down by half so that it will traverse the same space in double the time; and how could it be that these two half-speeds, which would be two rests, should together make the first speed? Finally cannot a space, however small, be cut in two, and its halves again cut? And how could it be that these halves should be indivisible without any extension, these halves which joined together made the first extension?

Man has no natural knowledge prior to this knowledge and surpassing it in clarity. Nevertheless, that there may be an example of everything, we find minds excellent in every other respect shocked by these infinites and in no way able to accept them.

I have never known a man who thought a space could not be increased. But I have come across some, and very clever too, who maintained that a space could be divided into two indivisible parts, whatever absurdity might be involved. I tried hard to find the cause of this darkness in them, and I discovered there was only one main cause, which is that they could not conceive an infinitely divisible content, whence they conclude that it is not infinitely divisible.

It is a disease natural to man to believe that he possesses the truth directly, and this is the reason he is always inclined to deny whatever he cannot understand. Whereas in fact he naturally knows nothing but error and should accept as true only those things whose contradictory appears to him to be false. Consequently, whenever a proposition is inconceivable, we must suspend our judgment and not deny it for that reason, but examine its contradictory; and if we find this manifestly false, we may boldly affirm the original statement, however incomprehensible it is. Let us apply this rule to our subject.

There is no geometer who does not believe that space is infinitely divisible. One can no more be a geometer without this principle than one can be a man without a soul. And yet there is no geometer who understands an infinite division. We are sure of that truth only for the reason, certainly sufficient, that we perfectly grasp the falsity of the statement that by dividing a space we can reach an indivisible part, a part, that is, having no extension.

For what is more absurd than to maintain that by continuing to divide a space we finally arrive at a division such that when we divide it in half, each of the halves remains indivisible and without any extension at all, and that thus these two nothings of extension should together make an extension? For I should like to ask those who have this idea whether they distinctly conceive of two indivisibles as touching. If they touch everywhere, they are only one self-same thing; and therefore the two together are indivisible. If they do not touch everywhere, they touch only in part; therefore they have parts; therefore they are not indivisible. But if they confess, as indeed they do when we press them, that their proposition is as inconceivable as the other, let them recognize that it is not by our capacity to conceive these things that we should judge of their truth, since though the two contradictories are both inconceivable, it is nevertheless necessarily certain that one of the two is true.

But to these imaginary difficulties, related only to our weakness, let them oppose these natural insights and solid truths: if it were true that space is composed of a certain finite number of indivisibles, it would follow that of two spaces, each a square (that is to say, having all its sides equal and similar) but one double the other, one would contain a number of these indivisibles double the number of the other. Let them keep this consequence well in mind, and then let them practise making squares out of points until they have found two of which one has double the points of the other, and then I shall make all the geometers in the world give place to them. But if this is naturally impossible, that is, if there is an invincible impossibility in making squares out of points of which one shall have double the points of the other, as I would demonstrate in this very place if it were worth the time, let them draw the conclusion.

And to comfort them in certain difficulties they would find, as in conceiving a space having an infinity of divisibles (in view of its being traversed in so short a time, during which time that infinity of divisibles would have been traversed), we must warn them that they must not compare things so disproportionate to each other as are the infinity of divisibles and the short time in which they are traversed, but that they should compare the whole of space with the whole of time, and the infinite divisibles of space with the infinite instants of time; and thus they will find we traverse an infinity of divisibles in an infinity of instants, and a little space in a little time, and in this there is no longer the disproportion that had overwhelmed them.

Finally, if it seems strange to them that a small space should have as many parts as a large space, let them also understand that they are proportionately smaller, and let them look at the heavens through a little pane of glass that they may grow familiar with this knowledge by seeing each part of the sky in each part of the glass. But if they cannot comprehend that parts so small as to be imperceptible to us can be divided into as many parts as can the heavens, there is no better remedy than to have them look at these delicate points through glasses which magnify each of them into a prodigious mass. Whence they will easily conceive that with the aid of another lens still more skilfully cut these points could be magnified to equal that firmament whose extent they wonder at. And thus these objects now appearing to them as very easily divisible, let them remember that nature is infinitely more powerful that art. For who has told them in fine that these lenses have changed the natural size of the objects or whether they have not on the contrary reëstablished their true size, changed and shrunk by the shape of our eye as by glasses that make smaller?

It is a nuisance to stop for these trifles, but there are times for being childish.

It is enough to say to heads that are clear in this matter that two nothings of extension cannot make an extension. But because there are some who claim to escape from the light of this truth by the wonderful reply that two nothings of extension can just as well make an extension as two units, neither of which is a number, can when brought together make a number, we must reply to them that they could in the same way object that 20,000 men make an army although no one of them is an army, that 1,000 houses make a city though no one of them is a city, or that the parts make the whole though no one of them is the whole, or, to stay within the comparison of numbers, that two dyads make a tetrad and ten decades a century though a dyad is not a tetrad and a decade is not a century. But it is muddleheaded to confuse by comparisons so unequal the unchanging nature of things with their names, which are free, arbitrary, and dependent upon the caprice of those who gave them. For it is clear that to facilitate discourse we have given the name of army to 20,000 men, that of city to several houses, that of decade to ten units, and that from this freedom arise the names *unit*, *dyad*, *tetrad*, *decade*, *century*, different by the dictates of fancy, although the things referred to are actually of the same kind by their invariable nature and are all related to each other in such a way that they differ only by more and less in spite of the fact that, because of their names, a dyad is not a tetrad, nor a house a city any more than a city is a house. But again although a house is not a city, it is not nevertheless a nothing of a city. There is a great difference between not being a thing and being a nothing of it.

For in order that we may understand the matter thoroughly, we must know that the only reason why the unit is not in the class of numbers is that Euclid and the first writers on arithmetic, having several properties to give belonging to all numbers but the unit, in order to avoid repeating that such and such a condition obtains for all numbers but the unit, excluded the unit from the meaning of the word *number* in accordance with the freedom we have already mentioned of making arbitrary definitions. Also if they had wanted to, they would have excluded the dyad and the triad as well, and anything else they pleased (for we are the masters, provided we give warning); as on the other hand the unit takes its place, if we choose, among the numbers, and the fractions too. And in fact in general propositions we are obliged to consider them there to avoid saying each time: "In every number and in the unit and in fractions such and such a property is found." It is in this indefinite sense I have taken number in all my writings about it. But the same Euclid who took away the name of number from the unit, which was his privilege, to make it clear nevertheless that the unit is not nothing but is on the contrary of the same kind, thus defines homogeneous magnitudes: "Magnitudes," he says, "are said to be of the same kind when one by being multiplied can be made to exceed the other." Consequently, since the unit can by being multiplied be made to exceed any number whatsoever, it is of the same kind as numbers precisely by its essence and unchanging nature, according to the same Euclid who would not have it called a number.

An indivisible with respect to an extension is a different matter; for not only does it differ in name, which is arbitrary, but it differs in kind by the above definition, since an indivisible multiplied as many times as we please is so far from being able to exceed an extension that it can never make anything but a single, unique indivisible, which is natural and necessary, as has already been shown. And since this last proof is based on the definition of these two things, indivisible and extension, we shall go on with the demonstration and complete it.

An indivisible is that which is without parts, and extension is that which has distinct separate parts.

On the basis of these definitions I say that two indivisibles when joined do not make an extension. For when they are joined each touches the other in a part; and thus the parts by which they are touching are not separate, since otherwise they would not be touching. However by definition they have no other parts; therefore they have no separate parts; therefore they are not an extension, by the definition which calls for separation of parts. The same thing can be shown by the same argument for all the other indivisibles joined to these. And therefore an indivisible, multiplied as many times as we please, will never make an extension. It is not then of the same kind as extension, by the definition of the same kind.

That is how we demonstrate that indivisibles are not of the same kind as extension. Hence it is that two units can indeed make a number since they are of the same kind, and that two indivisibles do not make an extension since they are not of the same kind. And therefore we see how little reason there is to compare the relation between the unit and numbers to that between indivisibles and extension.

But if we wish to find in numbers a comparison that will justly represent what we are considering in extension, we must take the relation of zero to the numbers, for zero is not of the same kind as numbers because it cannot by being multiplied be made to exceed them; so that zero is a true indivisible of number just as the indivisible is a true zero of extension. And we shall find a like relation between rest and motion, and between an instant and time; for they are all heterogeneous with respect to magnitude because though infinitely multiplied they can never make anything but indivisibles of extension, and for the same reason. And thus we shall find a perfect correspondence between these things; for all these magnitudes are infinitely divisible without ever being reduced to their indivisibles, so that all of them keep the middle ground between infinity and nothing.

Thus wonderfully has nature related these things, and such are the two marvelous infinites she has set before man, not for his mind's grasp but for its wonder; and to close our consideration of them with a last remark, I shall add that these two infinites, although infinitely different, are nevertheless related to each other in such a way that the knowledge of one necessarily leads to the knowledge of the other.

For in numbers, from the fact that they can always be increased, it follows absolutely that they can always be decreased, and that clearly. For if we can multiply a number up to 100,000 times, say, we can also take a hundred thousandth part of it by dividing it by the same number we multiply it with, and thus every term of increase will become a term of division by changing the integer into a fraction. So that infinite increase includes necessarily infinite division.

In space too we find the same relation between these two opposed infinites. That is to say, from the fact that a space can be infinitely extended, it follows that it can be infinitely diminished, as is apparent from the following example: if we watch through a window a ship moving straight away from us, it is clear that the place in the glass where we see any fixed point we please on the ship will keep rising by a continuous motion as the ship moves away. Therefore if the ship continues its course to infinity, the point will continue to rise, and yet it will never reach the foot of the horizontal line drawn from the eye to the glass, so that it will get closer and closer without ever reaching it, incessantly dividing the space which remains under the horizontal point without ever reaching it. Whence we see that from the infinite extent of the vessel's course there follows as a necessary consequence the infinite and infinitely small division of the little space remaining below the horizontal point.

Those who will not be satisfied with these reasons and continue in the belief that space is not infinitely divisible can never aspire to geometrical demonstrations, and although they may be enlightened in other things, they will have very little light in these, for it is easy to be a very clever man though a bad geometer. But those who clearly see these truths can admire the greatness and the power of nature in that double infinity which surrounds us on every side, and learn by the consideration of such marvels to know themselves, seeing themselves placed between an infinity and a nothing of extension, between an infinity and a nothing of number, between an infinity and a nothing of motion, between an infinity and a nothing of time. Whereupon we can learn to assess ourselves at our just value and to make reflections worth more than all the rest of geometry itself.

I felt obliged to consider at such length this double infinity for the sake of those who, though they do not understand it at first, can be persuaded of it. And although there are many who have light enough to do without this treatise, it may nevertheless be that, necessary to some, it will not be entirely useless to the rest.

SECTION II. *Concerning the art of persuasion*

The art of persuasion has a necessary relation to the way in which men agree to what we put before them and to the circumstances of the things we want them to believe.

We all know that opinions are admitted into the soul through two entrances, which are its two chief powers, understanding and will. The more natural entrance is the understanding, for we should never agree to anything but demonstrated truths, but the more usual entrance, although against nature, is the will; for all men whatsoever are almost always led into belief not because a thing is proved but because it is pleasing. This way is low, unworthy, and foreign to our nature. Therefore everybody disavows it. Each of us professes to give his belief and even his love only where he knows it is deserved.

I am not speaking here of divine truths, which I am far from bringing under the art of persuasion, for they are infinitely above nature. God alone can put them into the soul, and in whatever way He pleases. I know He has willed they should enter into the mind from the heart and not into the heart from the mind, that He might make humble that proud power of reason, which claims the right to be judge over the things chosen by the will, and that He might cure the infirmity of that will, which is all corruption from its unclean loves. Whence it comes about that, whereas in speaking of human things we say they must be known before they can be loved (which has passed into a proverb), the saints on the contrary say in speaking of divine things that they must be loved in order to be known, and that we enter into truth only through charity, out of which they have made one of their most useful maxims.

Whereby it is apparent that God established this order above nature and entirely opposed to the order which should be natural to men in natural things. Nevertheless they have corrupted this order by dealing with profane things as they ought to deal with sacred things, because actually we believe almost nothing unless it pleases us. And this is why we are so loath to accept the truths of the Christian religion, which is entirely opposed to our pleasures. "Tell us agreeable things and we will listen to you," the Jews said to Moses; as if agreeableness should govern belief! And it is to punish this disorder by an order conformable to it that God sheds His light into our minds only after having tamed the rebelliousness of the will by a heavenly sweetness which charms and seduces it.

I speak then only of truths within our reach, and it is of them that I say the mind and the heart are the gates, as it were, through which they are received into the soul; but very few enter through the mind, whereas they are brought in in crowds through the rash caprices of the will without the counsel of reason.

These powers have each their principles and prime motives of action.

Those of the mind are natural and universally known truths, such as the whole is greater than the part, besides many particular axioms accepted by some and not by others, which, though false, when once admitted, are as powerful to gain belief as are the truest.

The principles of the will are certain desires natural and common to all men, such as the desire to be happy, which it is impossible for anyone not to have, besides many particular objects which each pursues for the sake of happiness and which, although actually pernicious, because they are capable of pleasing us, are as strong to move the will as if they constituted its true happiness.

So much for the powers which induce our consent.

But as for the qualities of the things we are to persuade men of, they differ widely.

Some are drawn as necessary conclusions from common principles and accepted truths. Of these it is possible to persuade without fail, for by showing their relation to principles that have been granted it is inevitably necessary that we convince, and it is impossible for them not to be received into the soul from the moment we have been able to join them to those truths already admitted there.

There are others which are bound up with the objects of our satisfaction; and these too are received with certainty. For as soon as the soul is made to see that a thing can bring her to what she sovereignly loves, it is inevitable that she embrace it with joy.

But those which have a double bond, with admitted truths and with the heart's desires, are so certain of their effect that there is nothing more certain in nature. As on the other hand whatever is related neither to our beliefs nor to our pleasures is importunate to us, false, and completely foreign.

In all these cases there can be no doubt. But there are others where the things we wish to persuade of are firmly based on known truths, but at the same time are opposed to the pleasures which touch us most nearly. And these are in great danger of illustrating by an experience which is all too common what I said at the beginning: that this imperious soul, whose boast was to act only by reason, follows by a rash and shameful choice the desires of a corrupt will, whatever resistance the too enlightened mind may offer.

It is then that truth and pleasure hang doubtfully in the balance, and the knowledge of one and the feeling of the other engage in a struggle whose outcome is most uncertain since, to judge of it, we should have to know all that takes place in the innermost part of a man, which the man himself almost never knows.

Hence it is apparent that, no matter what we wish to persuade of, we must consider the person concerned, whose mind and heart we must know, what principles he admits, what things he loves, and then observe in the thing in question what relations it has to these admitted principles or to these objects of delight. So that the art of persuasion consists as much in knowing how to please as in knowing how to convince, so much more do men follow caprice than reason.

Now of these two, the art of convincing and the art of pleasing, I shall confine myself here to the rules of the first, and to them only in the case where the principles have been granted and are held to unwaveringly; otherwise I do not know whether there would be an art for adjusting the proofs to the inconstancy of our caprices.

But the art of pleasing is incomparably more difficult, more subtle, more useful, and more wonderful, and therefore if I do not deal with it, it is because I am not able. Indeed I feel myself so unequal to its regulation that I believe it to be a thing impossible.

Not that I do not believe there are as certain rules for pleasing as for demonstrating, and that whoever should be able perfectly to know and to practise them would be as certain to succeed in making himself loved by kings and by every kind of person as in demonstrating the elements of geometry to those who have imagination enough to grasp the hypotheses. But I consider, and it is perhaps my weakness that leads me to think so, that it is impossible to lay hold of the rules. At least I am sure that if anyone can, I know the persons, and that no one else has such clear and abundant light in this matter.

The reason for this extreme difficulty comes from the fact that the principles of pleasure are not firm and stable. They are different in all men, and they vary to such an extent in each individual that there is no man who differs more from another man than from himself at different times. A man has other pleasures than a woman has; a rich man and a poor man have different pleasures; a prince, a warrior, a merchant, a citizen, a peasant, the old, the young, the well, the sick, all vary in this respect; the slightest accidents change them.

However there is an art, the one I am going to expound, showing how truths are related to their principles, both of truth and of pleasure, provided the principles we have once admitted remain constant and are never denied.

But since there are few principles of this kind and since outside of geometry, which considers only very simple figures, there are almost no truths we always agree upon, and still fewer objects of pleasure we do not every hour change our mind about, I do not know whether it is possible to give fixed rules for making discourse conform to the inconstancy of our caprices.

The art which I call the art of persuading, and which is simply the management of perfect scientific proofs, consists of three essential parts: defining by clear definitions the terms to be used; laying down evident principles or axioms to prove the matter in question; always mentally substituting in the demonstration, in place of the things defined, their definitions.

The reason for this method is apparent, since it would be useless to put forward something capable of proof and to undertake its demonstration if we had not first clearly defined all unintelligible terms; and since likewise the demonstration must be preceded by the granting of the evident principles required for the demonstration, for if we do not make sure of the foundation, we can have no assurance of the building; and since finally while demonstrating we must mentally substitute the definition in place of the things defined, for otherwise we could be led astray by the different meanings encountered in the terms. It is easy to see that if we observe this method we are sure to convince, since, with all the terms so defined that they are understood and entirely free from ambiguity and with the principles granted, if in the demonstration we always substitute in thought the definitions in place of the things defined, the invincible force of the conclusions cannot fail of its full effect.

Accordingly no demonstration satisfying these conditions can ever have met with a shadow of doubt, and never can those demonstrations failing to satisfy them have any force.

It is very important, therefore, to understand and to appropriate them, and that is why, that they may be more easily grasped and more accessible, I shall give them all in these few rules which include everything necessary for the perfection of definitions, of axioms, and of demonstrations, and consequently of the entire method of geometrical proofs of the art of persuasion.

Rules for definitions. 1. Do not attempt to define any of those things so well known in themselves that we have no clearer terms to explain them by.

2. Do not leave undefined any terms that are at all obscure or ambiguous.

3. Use in the definition of terms only words perfectly well known or already explained.

Rules for axioms. 1. Do not fail to ask that each of the necessary principles be granted, however clear and evident it may be.

2. Ask only that perfectly self-evident things be granted as axioms.

Rules for demonstrations. 1. Do not attempt to demonstrate any of those things so self-evident that we have nothing clearer to prove them by.

2. Prove all propositions that are at all obscure, and use for their proof only axioms that are perfectly evident or propositions already granted or demonstrated.

3. Always substitute mentally the definitions in place of the things defined, so you will not be led astray by the equivocation of terms whose meanings have been restricted by the definitions.

These are the eight rules which contain the precepts for firm and abiding proofs, three of which are not absolutely necessary and can be neglected without error. It is even difficult and impossible, as it were, always to observe them exactly, although for the sake of perfection we should do so to the best of our ability. These three are those which come first in each of the divisions:

For the definitions. Do not define any of the terms that are perfectly well known.

For the axioms. Do not fail to ask that all the perfectly evident and simple axioms be granted.

For the demonstrations. Do not demonstrate anything very well known in itself.

For it is undoubtedly not a great fault to define and very clearly to explain things which are most clear in themselves, nor to fail to ask in advance that axioms be granted which no one can refuse to grant when the necessity for their use arises, nor finally to prove propositions which would be granted without proof.

But the five other rules are absolutely necessary, and we cannot dispense with them without an essential defect and frequently even error. That is why I shall repeat them here in detail.

Necessary rules for definitions. Do not leave undefined any terms at all obscure or ambiguous. Use in definitions only terms perfectly well known or already explained.

Necessary rule for axioms. Ask only that evident things be granted as axioms.

Necessary rules for demonstrations. Prove all propositions, using for their proof only axioms that are perfectly self-evident or propositions already demonstrated or granted. Never get caught in the ambiguity of terms by failing to substitute in thought the definitions which restrict or explain them.

These five rules comprise everything necessary for making proofs convincing, unchanging, and in a word, geometrical; and the eight together make them still more perfect.

I pass now to the order in which propositions should be arranged if they are to have the right geometrical sequence.

After having established . . .

These are the rules of the art of persuasion, which are contained in these two principles: Define all the names imposed; prove everything, substituting in thought the definitions in place of the things defined.

Whereupon it seems to me in order to anticipate three chief objections that can be made: first, that this method is not at all new; second, that it is very easy to learn without there being any necessity of studying the elements of geometry, since it consists of these two rules that are known as soon as read; and third, that it is pretty useless since its use is almost confined to the objects of geometry alone.

It must then be shown that there is nothing so little known, nothing harder to practise, and nothing more useful and more universal.

As for the first objection, which is that these rules—everything must be defined and everything proved—are common knowledge in the world and that the logicians themselves have included them among the precepts of their art, I only wish it were true and so well known that I should not have had the labor of seeking with such care for the source of all the defects of reasoning, which are really common. But this is so little the case that with the single exception of geometers, who are so few in number as to be unique among a whole people and over a long period of time, we find no one who knows these rules. It will be easy to make this clear to those who have grasped perfectly the little I have said about them; but if they have not perfectly understood that, I admit they will have nothing to learn in this matter. But if these rules have found a way into their minds and have made a strong enough impression to take root and flourish there, they will feel how great a difference there is between what is said in this place and such approximations to it as some logicians may have made by chance here and there in their works.

Discerning minds know how much difference there is between two similar remarks, depending upon the place and accompanying circumstances. Will anyone really believe that two persons who have read and learned by heart the same book know it equally well, if one understands it in such a way that he knows all its principles, the force of its conclusions, the replies to the objections that can be made, and the entire organization of the work, whereas in the other the book is dead words and seeds which, though the same as those that produced such fertile trees, have remained dry and unfruitful in the sterile mind which received them in vain?

Not all those who say the same things possess them in the same way. And that is why the incomparable author of the *Art of Conversation*[1] is at such pains to make it clear that we must not judge a man's capacity by the excellence of an observation we have heard him make. Instead of carrying over to the speaker our admiration of the speech, we should, he says, penetrate the mind it comes from, try and find if he speaks by memory or by a happy chance, listen coldly and scornfully to see if he will resent our not having for what he says the esteem it deserves; we shall find more often than not that he will disavow it on the spot and that, led away from a thought whose worth he does not know, he will fall into another thoroughly common and ridiculous. We must then fathom how that thought is lodged in its author; how, whence, and to what extent he has made it his own; otherwise the judgment, acting in haste, will be judged rash.

I should like to ask fair-minded persons if this principle: "Matter is naturally and invincibly incapable of thought," and this other: "I think, therefore I am," are actually the same in the mind of Descartes and in the mind of St. Augustine, who said the same thing 1200 years before.

Certainly I am very far from saying that Descartes is not its true author, even if he should have learned it only from the reading of that great saint; for I know how much difference there is between making a statement off hand, without considering it more fully and at length, and seeing in that statement an admirable succession of consequences, which proves that matter is distinct from mind, and making out of it a firm and consistently held principle of a whole physics, as it was the intention of Descartes to do. For without examining whether he was successful in carrying out his intention, I assume that he was, and it is on this assumption that I say the statement in his writings is as different from the same statement in those others who have made it in passing as a man full of life and force is different from a dead man.

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[1] Montaigne, *Essays*, III, 8.

One man will say a thing off hand without understanding its excellence where another will see a wonderful succession of consequences which make us say boldly that it is no longer the same observation, and that he no more owes it to him from whom he learned it than a splendid tree belongs to some one who scattered the seed, thoughtlessly and without knowing what it was, in a rich soil which derived this profit from the seed by its own fertility.

The same thoughts sometimes shoot up in another quite otherwise than in their author, unfertile in their native field, abundant when transplanted. But it happens much more often that a good mind itself makes its own thoughts bear all the fruit they are capable of, and that afterwards others, having heard them praised, borrow them and deck themselves out with them, but without knowing their worth; and it is then that the difference of the same observation in different mouths is most apparent.

It is in this fashion that logicians may have borrowed the rules of geometry without understanding their force, and thus it does not follow from their happening to include them among the rules proper to logic that they have entered into the spirit of geometry; and unless they give me other indications of it than the passing mention of these rules, I shall be very far from putting them on the level of that science, which teaches the true method for the conduct of reason. But on the contrary I shall be very much inclined to exclude them, and almost without hope of return. For to have mentioned these rules in passing without noticing that everything is contained in them, and instead of following their light to wander aimlessly in useless investigations, running after what they offer and cannot give, is certainly to show oneself as hardly clearsighted and as having failed to follow the light because one had not seen it.

The method of avoiding error is sought by everyone. The logicians profess to guide us to it, only the geometers reach it, and outside of their science and its imitators there are no true demonstrations. The entire art is contained in these precepts we have given, they alone are sufficient, they alone prove; all other rules are useless or harmful. This is what I have learned from a long experience of all kinds of books and men.

Wherefore I judge those who say that geometers give them nothing new by these rules because they actually had them but mixed up with a lot of others, useless or false, from which they could not separate them out, I judge them, I say, as I do those who, seeking a diamond of great price among many that are false, which they however could not tell from it, should boast, holding them all together, that they possess the diamond as truly as he who, without stopping at the worthless mass, puts his hand on the choice stone they are seeking and for which they did not throw away all the rest.

The defect of false reasoning is a malady which is cured by these two remedies. Another has been compounded of many useless herbs among which the good are mingled but remain without effect because of the bad qualities of the mixture.

To uncover all the sophisms and equivocations of captious arguments they have invented barbarous names which astonish those who hear them; and whereas all the windings of this tangled knot can be straightened out only by pulling one of the ends designated by the geometers, they have noted a monstrous number of others in which these are included, without their knowing which are the right ones.

And thus those who point out to us a number of different roads, which they say will take us where we want to go, although only two of them lead there, must be able to give them a particular mark. It will be said that geometry, which indicates the roads with certainty, only gives what had already been given by the others, because they actually gave the same thing and more, without its being seen that this present lost its value by its very abundance and that by adding they took away.

Nothing is more common than good things; the only question is how to discern them; it is certain that all of them are natural and within our reach and even known by every one. But we do not know how to distinguish them. This is universal. It is not in things extraordinary and strange that excellence of any kind is found. We reach up for it, and we are further away; more often than not we must stoop. The best books are those whose readers think they could have written them. Nature, which alone is good, is familiar and common throughout.

I have no doubt then that these rules, because they are the true ones, should be simple, naive, natural, as they are. It is not *barbara* and *baralipton* which form the reason. We must not force the mind to fly too high. Strained and painful behavior fills it with foolish presumption by giving it an elevation foreign to it and an empty and ridiculous inflation instead of solid and vigorous nourishment. And one of the main reasons which keep those who are beginning these studies out of the true road they ought to follow is the notion they get at the start that the good things are inaccessible because they bear the names: great, high, exalted, sublime. That spoils everything. I should like to call them humble, common, familiar. These names suit them better. I hate those inflated words. . . .